

# A Dispersion Formula Satisfying Recent Requirements in Microstrip CAD

MASANORI KOBAYASHI, MEMBER, IEEE

**Abstract** — A dispersion formula,  $\epsilon_{\text{eff}}^*(f) = \epsilon^* - \{\epsilon^* - \epsilon_{\text{eff}}^*(0)\}/\{1 + (f/f_{50})^m\}$ , for the effective relative permittivity  $\epsilon_{\text{eff}}^*(f)$  of an open microstrip line is derived satisfying recent CAD requirements. Closed-form computations with error less than 1 percent compared with numerical solutions are obtained. The frequency  $f_{50}$  at which  $\epsilon_{\text{eff}}^*(f_{50}) = \{\epsilon^* + \epsilon_{\text{eff}}^*(0)\}/2$  (the 50 percent dispersion point) is used as a normalizing frequency in the proposed formula, and an expression for  $f_{50}$  is derived. In order to obtain the best fit of  $\epsilon_{\text{eff}}^*(f)$  to the theoretical numerical model, the power  $m$  of the normalized frequency in the proposed formula is expressed as a function of  $w/h$  for  $w/h \geq 0.7$  and as a function of  $w/h$ ,  $f_{50}$ , and  $f$  for  $w/h \leq 0.7$ . The present formula has a high degree of accuracy, better than 0.6 percent in the range  $0.1 < w/h \leq 10$ ,  $1 < \epsilon^* \leq 128$ , and any  $h/\lambda_0$ .

## I. INTRODUCTION

MICROSTRIP transmission lines are an essential part of integrated circuits, from which modern microwave components are made. Their dispersion properties have been studied theoretically by a number of researchers ([1]–[13] and references therein). Kuester and Chang [1] pointed out that significant discrepancies are found in effective relative permittivities  $\epsilon_{\text{eff}}^*(f)$  computed from different sources.

On the other hand, many approximate dispersion formulas have been proposed to provide information on the dispersion behavior of microstrip. The validity of each of these formulas has been checked by comparing their results with the theoretical ones with the discrepancies between them mentioned in [1] and [13]. The major cause of the discrepancies is that many of the theoretical results were calculated with a small number of basis functions for current distributions to save CPU time, although the methods are intrinsically rigorous. What is more, the current distributions were not expressed accurately, as explained in [13]. Therefore, each formula may have its own range of validity but none is valid over a broad range of frequency and/or physical parameters. However, the results over a broad range tabulated in [13] were calculated by using simple but accurate closed-form expressions for the current distributions. Therefore they have a high degree of accuracy and can be used to estimate the accuracy of many formulas.

CAD requires accurate and reliable information on the dispersion behavior of microstrip. For example, it is said that for direct application in microstrip CAD programs [2]

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The author is with the Department of Electrical Engineering, Faculty of Engineering, Ibaraki University, Hitachi, Ibaraki, Japan.

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the dispersion formula for  $\epsilon_{\text{eff}}^*(f)$  should exhibit an error of less than 1 percent compared to rigorous numerical solutions. Kirschning and Jansen [2] compared the results of the closed-form expressions published at the time with the numerical results. Those expressions were valid at low frequencies but had errors at high frequencies which were not tolerable. Therefore, they proposed a closed-form expression with a high degree of accuracy [2].

The distortion of an electric pulse caused by dispersion as it propagates along a microstrip line has also been investigated [3]. Frequency components up to  $f = 100$  to 200 GHz were found for  $h/\lambda_0 = 2.1$  to 4.2 with  $h = 0.025$  in. [3]. An analysis for its distortion requires an accurate dispersion formula for  $\epsilon_{\text{eff}}^*(f)$  over such frequency and parameter ranges.

A number of dispersion formulas have been subsequently derived by modifying many of the previously available formulas published, as explained in the present paper. Now, we can estimate the accuracy of each formula by comparing it with a numerical solution [13]. It shows that a more accurate dispersion formula seems to be needed.

In this paper, a dispersion formula for  $\epsilon_{\text{eff}}^*(f)$  of an open microstrip line is derived that satisfies both requirements presented in [2] and [3].

## II. DISPERSION FORMULAS

Fig. 1 shows an open microstrip line structure. The infinitesimally thin strip and the ground plane are considered perfect conductors. It is also assumed that the substrate material is lossless and that its relative permittivity and permeability are  $\epsilon^*$  and  $\mu^*$  ( $= 1$ ), respectively.

Schneider [5] has found that the second-order derivative of the normalized phase velocity with respect to the frequency is zero in the vicinity of the cutoff frequencies  $f_c$  of the TE<sub>1</sub> surface-wave mode. Using this cutoff frequency as the normalizing frequency, he proposed a dispersion formula for the effective relative permittivity  $\epsilon_{\text{eff}}^*(f)$  given by

$$\frac{1}{\sqrt{\epsilon_{\text{eff}}^*(f)}} = \frac{1}{\sqrt{\epsilon^*}} - \frac{\frac{1}{\sqrt{\epsilon^*}} - \frac{1}{\sqrt{\epsilon_{\text{eff}}^*(0)}}}{1 + (f/f_c)^2} \quad (1)$$

where  $f_c = c/(4h\sqrt{\epsilon^* - 1})$  and  $c$  is the velocity of light in free space. It was noted that this formula required some

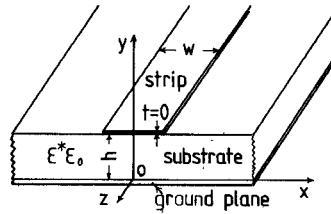


Fig. 1. Microstrip configuration.

modifications [11], [12]. Nevertheless it contributed to the further development of improved expressions of  $\epsilon_{\text{eff}}^*(f)$ .

A year later, Getsinger [6] proposed a modified dispersion formula:

$$\epsilon_{\text{eff}}^*(f) = \epsilon^* - \frac{\epsilon^* - \epsilon_{\text{eff}}^*(0)}{1 + \left\{ f / \left( f_p / \sqrt{G} \right) \right\}^2} \quad (2)$$

where  $G = 0.6 + 0.009Z_0$ ,  $f_p = Z_0 / (2\mu_0 h)$ ,  $Z_0$  is the characteristic impedance at zero frequency, and  $\mu_0$  is the free-space permeability. However, there are misprints in figure 1 and equation (26) of [6], and the following corrections should be made:  $(3\epsilon_s - \epsilon_{e0})/2 \rightarrow (G\epsilon_s + \epsilon_{e0})/(1+G)$  in figure 1 and  $3(\epsilon_s - \epsilon_{e0})/8 \rightarrow 3(\epsilon_s - \epsilon_{e0})/(8f)$  in equation (26). The present article uses the idea shown in figure 1 of [6] in considering the dispersion formula.

Edwards and Owens [7] modified  $G$  as follows:

$$G = \left( \frac{Z_0 - 5}{60} \right)^{1/2} + 0.004Z_0. \quad (3)$$

Furthermore, Hammerstad and Jensen [8] modified  $G$  as

$$G = \frac{\pi^2}{12} \frac{\epsilon^* - 1}{\epsilon_{\text{eff}}^*(0)} \sqrt{\frac{2\pi Z_0}{\eta_0}} \quad (4)$$

where  $\eta_0 = \sqrt{\mu_0 / \epsilon_0}$  is the wave impedance in free space.

Recently, Pramanick and Bhartia [4] modified  $G$  as

$$G = \epsilon_{\text{eff}}^*(0) / \epsilon^*. \quad (5)$$

On the other hand, Edwards and Owens [7] expressed the dispersion formula in its more general form as

$$\epsilon_{\text{eff}}^*(f) = \epsilon^* - \frac{\epsilon^* - \epsilon_{\text{eff}}^*(0)}{1 + P(f)} \quad (6)$$

where

$$P(f) = (h/Z_0)^{1.33} (0.43f^2 - 0.009f^3)$$

with  $h$  in mm and  $f$  in GHz.

Kirschning and Jansen [2] modified  $P(f)$  as

$$P(f) = P_1 P_2 [(0.1844 + P_3 P_4) 10 f h]^{1.5763} \quad (7)$$

where  $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$  are given in [2].

Yamashita *et al.* [9] expressed the dispersion formula in the form

$$\frac{1}{\sqrt{\epsilon_{\text{eff}}^*(f)}} = \frac{(f/f_Y)^{1.5} + 4}{(f/f_Y)^{1.5} \sqrt{\epsilon^* + 4\sqrt{\epsilon_{\text{eff}}^*(0)}}} \quad (8)$$

TABLE I  
PERCENTAGE ERROR OF THE DISPERSION FORMULA (KIRSCHNING AND JANSEN [2]) FOR  $\epsilon_{\text{eff}}^*(f)$

$\epsilon^*$	$h/\lambda_0$	0.005	0.05	0.1	0.2	0.3	0.4	0.7	1.0	maximum
2	0.4	*	-0.2	-0.2	0.2	0.3	0.2	-0.1	-0.5	0.5
	1	*	-0.2	*	0.4	0.6	0.6	0.4	0.3	0.6
	2	*	*	0.3	0.7	0.8	0.8	0.6	0.5	0.8
4	0.4	*	-0.5	-0.3	-0.1	-0.5	-0.8	-1.2	-0.7	1.2
	1	*	-0.4	*	0.3	0.3	0.3	0.1	*	0.4
	2	*	-0.2	0.3	0.8	0.9	0.8	0.5	0.3	0.9
8	0.1	*	*	0.4	-1.7	-3.2	-3.3	-2.1	-1.4	3.3
	0.4	*	*	0.3	-0.4	-1.0	-1.2	-0.9	-0.6	1.2
	1	0.1	*	0.5	0.5	0.3	0.2	*	*	0.5
16	2	0.2	0.2	1.0	1.2	0.9	0.7	0.3	0.2	1.2
	10	0.2	1.3	1.4	0.9	0.6	0.5	0.3	0.2	1.4
	0.4	*	0.4	0.2	-0.6	-1.2	-1.0	-0.6	-0.4	1.2
128	1	0.2	0.5	0.7	0.4	0.1	*	-0.1	-0.1	0.7
	2	0.3	0.8	1.4	1.0	0.6	0.4	0.2	*	1.4
	0.4	-1.6	10.1	-7.0	-2.9	-1.7	-1.2	-0.5	-0.3	10.1
256	1	-1.9	6.2	-3.4	-1.5	-0.9	-0.3	-0.1	-0.6	6.2
	2	-2.7	-3.1	-1.3	0.6	0.2	-0.1	*	-0.3	3.1

obtained as compared to a numerical solution [13].  
\* : less than 0.09.

where

$$f_Y = \frac{c}{4h\sqrt{\epsilon^* - 1} \left[ 0.5 \left\{ 1 + 2\log_{10} \left( 1 + \frac{w}{h} \right) \right\}^2 \right]}.$$

Of these formulas, we now select those by Pramanick and Bhartia [4], Kirschning and Jansen [2], and Yamashita *et al.* [9], taking into account the restrictions discussed in the Introduction. The case of  $\epsilon^* = 8$  and  $w/h = 1$  was calculated by these formulas. The percentage errors as compared to a numerical solution [13] are obtained. These results showed that the formula (KJ) by Kirschning and Jansen [2] is the most accurate one. Therefore, the other cases were calculated by using the formula KJ. Table I shows those results. It is said in [2] that the formula KJ has an accuracy of better than 0.6 percent in the range  $0.1 \leq w/h \leq 100$ ,  $1 \leq \epsilon^* \leq 20$ , and  $0 \leq h/\lambda_0 \leq 0.13$ . This can be confirmed for the cases of  $w/h \leq 1$  by the results shown in Table I. It is evident from the table that a more accurate dispersion formula is needed to satisfy the accuracy requirements (for example, [2] and [3]) for microstrip CAD.

### III. ACCURATE DISPERSION FORMULA

The present article proposes a new dispersion formula for  $\epsilon_{\text{eff}}^*(f)$ :

$$\epsilon_{\text{eff}}^*(f) = \epsilon^* - \frac{\epsilon^* - \epsilon_{\text{eff}}^*(0)}{1 + (f/f_{50})^m} \quad (9)$$

where

$$f_{50} = \frac{f_{K,\text{TM}_0}}{0.75 + \left( 0.75 - \frac{0.332}{\epsilon^{1.73}} \right) \frac{w}{h}} \quad (10)$$

$$f_{K,\text{TM}_0} = \frac{c \tan^{-1} \left\{ \epsilon^* \sqrt{\frac{\epsilon_{\text{eff}}^*(0) - 1}{\epsilon^* - \epsilon_{\text{eff}}^*(0)}} \right\}}{2\pi h \sqrt{\epsilon^* - \epsilon_{\text{eff}}^*(0)}} \quad (11)$$

TABLE II  
PERCENTAGE ERROR OF THE PRESENT DISPERSION FORMULA FOR  
 $\epsilon_{\text{eff}}^*(f)$

$\epsilon^*$	$h/\lambda_0$	0.005	0.05	0.1	0.2	0.3	0.4	0.7	1.0	maximum
2	0.4	*	-0.2	-0.2	*	0.2	0.3	0.5	0.1	0.5
	1	*	-0.3	-0.3	-0.2	*	*	*	*	0.3
	2	*	-0.2	-0.3	-0.3	-0.2	*	*	*	0.3
4	0.4	*	-0.3	-0.1	0.2	0.2	0.2	-0.1	0.2	0.3
	1	*	-0.4	-0.2	-0.1	*	*	*	-0.1	0.4
	2	*	-0.1	-0.2	*	0.1	0.1	*	*	0.2
8	0.1	*	*	1.2	-0.3	-0.7	-0.5	*	0.1	1.2
	0.4	*	-0.2	0.1	*	*	*	0.1	*	0.2
	1	*	-0.3	-0.1	*	-0.1	-0.2	-0.2	-0.2	0.3
	2	*	-0.1	*	0.2	0.2	*	*	*	0.2
	10	-0.2	0.5	0.6	0.4	0.2	0.2	*	*	0.6
16	0.4	*	*	*	-0.3	-0.2	-0.1	*	*	0.3
	1	*	-0.2	*	-0.2	-0.3	-0.3	-0.3	-0.2	0.3
	2	0.1	*	0.3	0.2	*	*	*	*	0.3
128	0.4	-0.2	-0.2	-0.3	*	*	*	*	*	0.3
	1	-0.3	*	-0.3	-0.3	-0.2	-0.2	-0.1	*	0.3
	2	-0.3	0.4	*	-0.1	-0.1	-0.1	*	*	0.4
obtained as compared to a numerical solution [13].										
* : less than 0.09.										

$$m = m_0 m_c (\leq 2.32) \quad (12)$$

$$m_0 = 1 + \frac{1}{1 + \sqrt{w/h}} + 0.32 \left( \frac{1}{1 + \sqrt{w/h}} \right)^3 \quad (13)$$

$$m_c = \begin{cases} 1 + \frac{1.4}{1 + \frac{w}{h}} \left\{ 0.15 - 0.235 \exp \left( \frac{-0.45f}{f_{50}} \right) \right\} & (w/h \leq 0.7) \\ 1 & (w/h \geq 0.7). \end{cases} \quad (14)$$

$$(w/h \geq 0.7). \quad (15)$$

Table II shows the percentage error of the present formula, (9), compared to a numerical model [13]. It is seen in Table II that the present formula has a high degree of accuracy, better than 0.6 percent in the range  $0.1 < w/h \leq 10$ ,  $1 < \epsilon^* \leq 128$ , and any  $h/\lambda_0$ .

Calculating  $\epsilon_{\text{eff}}^*(f)$  by using the present formula, the only unknown value is  $\epsilon_{\text{eff}}^*(0)$ , the effective relative permittivity at  $f = 0$ . This  $\epsilon_{\text{eff}}^*(0)$  can be obtained by using an effective filling fraction  $q_w$  as follows [10]:

$$\epsilon_{\text{eff}}^*(0) = 1 + q_w (\epsilon^* - 1). \quad (16)$$

The values of  $q_w$  for various cases were given with a very high degree of accuracy in table I of [10]. The values of  $q_w$  for the cases not given in that table can be obtained by an interpolation method using the tabulated values.

On the other hand, an accurate approximate formula for  $q_w$  was already published which has an accuracy better than 0.2 percent [14]. For the cases with substrate of  $\epsilon^* = 1.01, 2$ , or  $8$ ,  $q_w$  can be straightforwardly calculated by using equation (7) or (8) of [14]. For the cases of other  $\epsilon^*$ ,  $q_w$  can be calculated by using equation (6) of [14] although a little complicated computation is needed.

As an example,  $q_w$  will be calculated for the case of  $w/h = 10$ ,  $\epsilon^* = 16$ , and  $h = 1$  mm.  $q_w$  is a function of  $WH$  ( $= w/h$ ) and  $\epsilon^*$ ;  $q_w(WH, \epsilon^*)$ . First, we can obtain

$q_w(10, 1.01) = 0.854216$ ,  $q_w(10, 2) = 0.847702$ , and  $q_w(10, 8) = 0.841167$  by substituting the values of  $WH$ , and  $q_w(WH, \epsilon^*)$  given in table I of [14] and the values of  $b_1$  and  $b_2$  given in table II of [14] into equation (7) of [14], respectively. Next, we can obtain  $a_0 = 0.854324$ ,  $a_1 = -0.010853$ , and  $a_2 = -0.004778$  by solving the simultaneous equations obtained by substituting the values of  $q_w(10, 1.01)$ ,  $q_w(10, 2)$ , and  $q_w(10, 8)$  obtained above into equation (6) of [14], respectively. We can obtain  $q_w(10, 16) = 0.83995$  accurately by using equation (6) of [14] with these values for  $a_0$ ,  $a_1$ , and  $a_2$ . An error of about 0.032 percent is obtained with respect to the exact value  $q_w = 0.839679$  given in table I of [10].

Substituting this  $q_w(10, 16)$  into (16), we can obtain  $\epsilon_{\text{eff}}^*(0) = 13.599$ . Using this  $\epsilon_{\text{eff}}^*(0)$ ,  $w/h$ ,  $\epsilon^*$ , and  $h$ , we obtain subsequently  $f_{K, \text{TM}_0} = 47.529$  GHz,  $f_{50} = 5.7803$  GHz,  $m_c = 1$ ,  $m_0 = 1.2447$ , and  $m = 1.2447$  from (11), (10), (15), (13), and (12), respectively. Using (9) with these values, we can obtain straightforwardly  $\epsilon_{\text{eff}}^*(f) = 13.842$ , 14.691, 15.578, 15.802, and 15.909 for  $f = 1, 5, 10, 20, 40$ , and 80 GHz, respectively.

#### IV. BACKGROUND FOR A HIGH DEGREE OF ACCURACY WITH THE PRESENT FORMULA

The author has previously formulated different approximate formulas and discussed the important role of the inflection frequency  $f_i$  on the dispersion curves [11], [12]. He tried to obtain, with a high degree of accuracy, the inflection frequencies  $f_i$  for various cases by using numerical models [13]. However, it was difficult to localize accurately the inflection point on the dispersion curve. On the other hand, it was comparatively easy to find the point at which

$$\epsilon_{\text{eff}}^*(f_{50}) = \{\epsilon^* + \epsilon_{\text{eff}}^*(0)\}/2. \quad (17)$$

Let this point be called the 50 percent dispersion point and let the frequency at this point be denoted by  $f_{50}$ .

The present article accurately obtained  $f_{50}$  in place of  $f_i$  for the various cases. To give an optimum fit to these values, expression (10) for  $f_{50}$  was derived.

Fig. 2 shows the influence of the power  $m$  on the dispersion curves for the case of  $\epsilon^* = 8$  and  $w/h = 1$ . In Fig. 2, the solid lines denote the dispersion curves from formula (9) with  $f_{50}$  given by (10); the dotted line denotes the theoretical numerical results [13], and the dot-dash line the dispersion curve of the  $\text{TM}_0$  surface wave mode [15]. Fig. 2 shows that  $m = 1.54$  obtained from (12) gives a best fit to the theoretical curve.

We can easily show by comparison with a numerical model of  $\epsilon_{\text{eff}}^*(f)$  [13] and  $\text{TM}_0$  dispersion values  $\epsilon_{\text{eff}, \text{TM}_0}^*$  that the theoretical curves are always above those of the  $\text{TM}_0$  mode and are asymptotic at high frequencies. Fig. 3 illustrates this fact. In this figure, the dots denote the theoretical results and the dot-dash line the  $\text{TM}_0$  dispersion curve for the case of  $\epsilon^* = 8$  and  $w/h = 0.1$ . We found that a constant  $m$  cannot give a good fit to the theoretical curves when  $w/h < 0.7$ . Smaller  $m$  and larger  $m$  were required below and above the  $f_{50}$  point, respectively. This

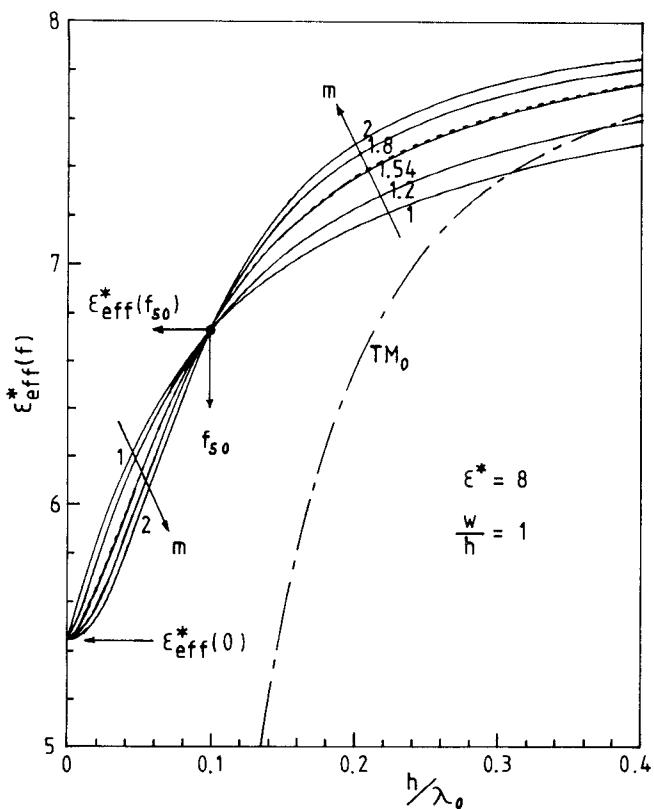


Fig. 2. Influence of the power  $m$  of frequency  $f$  on the dispersion curves.  $\epsilon^* = 8$ ,  $w/h = 1$ . — present dispersion formula (9) with  $f_{50}$  of (10). - - - theoretical results [13]. —  $\text{TM}_0$  surface wave mode

requirement is satisfied by letting  $m$  be expressed as a function not only of  $w/h$  but also of  $f$  and  $f_{50}$  when  $w/h < 0.7$ . The  $m$  of (12) with  $m_c$  given by (14) satisfies this requirement; the upper limit 2.32 of  $m$  prevents the dispersion curve from going largely above the  $\text{TM}_0$  curve at high frequencies for smaller  $w/h$ . The double dotted-dash line in Fig. 3 illustrates the validity of this expression. To show this, Fig. 3 illustrates, with a solid line, the dispersion curve given by (9) with  $m = m_0$  ( $= \text{constant}$ ). It also shows the dispersion curves for the case of  $w/h = 0.7$ .

It is meaningful from practical and educational points of view to illustrate the dispersion characteristics for various  $\epsilon^*$  and  $w/h$  in one figure. The dispersion characteristics for the cases of  $\epsilon^* = 8$  and various  $w/h$  are computed by the present formula and are illustrated in Fig. 4. The portion of the curves below the  $\text{TM}_0$  mode curve are not shown in Fig. 4.

No one can dispute the fact that for the curve of  $\epsilon_{\text{eff}}^*(f)$  for "isotropic" dielectric substrates, a horizontal line  $\epsilon_{\text{eff}}^*(f) = \epsilon_{\text{eff}}^*(0)$  is a lower bound at low frequencies and the dispersion curve for the  $\text{TM}_0$  mode is a lower bound at high frequencies. The quantity  $f_{K,\text{TM}_0}$  of (11) is the frequency at the intersection point of this horizontal line and the  $\text{TM}_0$  curve. Therefore, the author guessed that this intersection point may have an important role in the dispersion characteristics. At the present time, he cannot say more than this about why  $f_{50}$  was related to  $f_{K,\text{TM}_0}$ .

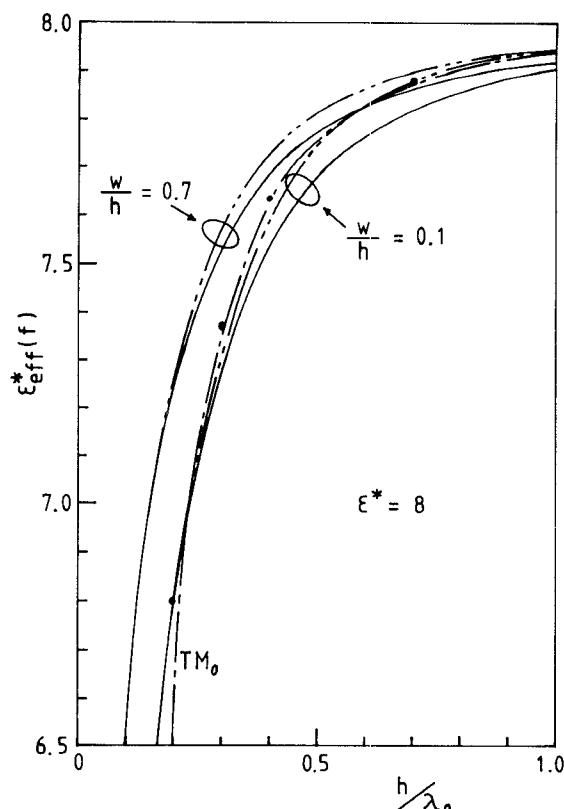


Fig. 3 Relations among the dispersion curves by formula (9) with  $m = m_0$  ( $= \text{constant}$ ) and with  $m$  of (12) with  $m_c$  of (14), the  $\text{TM}_0$  dispersion curve, and the theoretical results  $\epsilon^* = 8$ ,  $w/h = 0.1$  and 0.7 — dispersion formula (9) with  $m = m_0$  ( $= \text{constant}$ ) — — dispersion formula (9) with  $m$  of (12) with  $m_c$  of (14) — —  $\text{TM}_0$  surface wave mode • theoretical results [13] for  $w/h = 0.1$

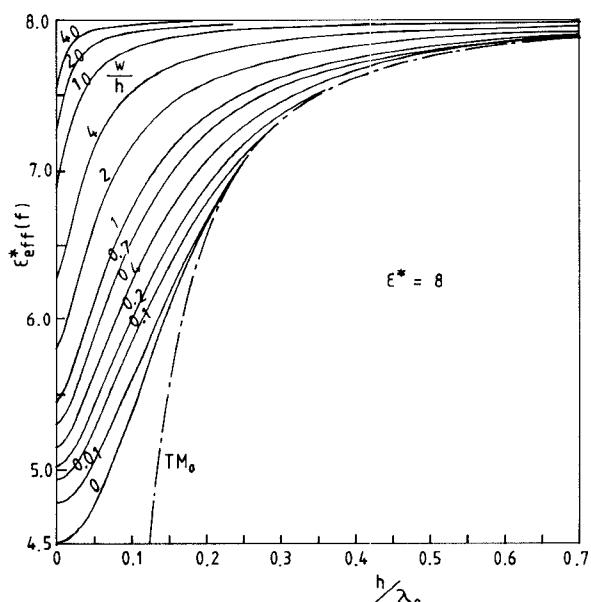


Fig. 4. Dispersion characteristics for the cases of  $\epsilon^* = 8$  and various  $w/h$ . — the present dispersion formula (9) — —  $\text{TM}_0$  surface wave mode

But one should observe the high degree of accuracy of the present dispersion formula.

## V. CONCLUSIONS

It has been shown that a number of available dispersion formulas for  $\epsilon_{\text{eff}}^*(f)$  have been progressively derived by modifying many of the previously available expressions. By comparing the results of three of the available formulas with a numerical solution it has been shown that a more accurate dispersion formula is needed. The present formula, derived by comparison to a numerical model, has a high degree of accuracy, better than 0.6 percent in the range  $0.1 < w/h \leq 10$ ,  $1 < \epsilon^* \leq 128$ , and any  $h/\lambda_0$ . This accuracy satisfies recent requirements in microstrip CAD.

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**Masanori Kobayashi** (M'79) was born in Niigata, Japan, on June 17, 1947. He received the B.E. and M.E. degrees in electrical engineering from Ibaraki University, Ibaraki, Japan, in 1970 and 1972, respectively, and the D.E. degree in electrical and electronic engineering from the Tokyo Institute of Technology, Tokyo, Japan, in 1981.

Since April 1972 he has been with the Department of Electrical Engineering at Ibaraki University. He was a Research Assistant from April 1972 to March 1981 and a Lecturer from April 1981 to July 1982. Since August 1982, he has been an Associate Professor. His research interests are in the areas of microstrip transmission lines, dielectric optical waveguides, magnetic elements, and relativistic electromagnetic theory.

Dr. Kobayashi is a member of the Institute of Electrical Engineers of Japan and the Institute of Electronics, Information and Communication Engineers of Japan.